Time Delay Interferometry with USO Noise

Daniel

August 7, 2002

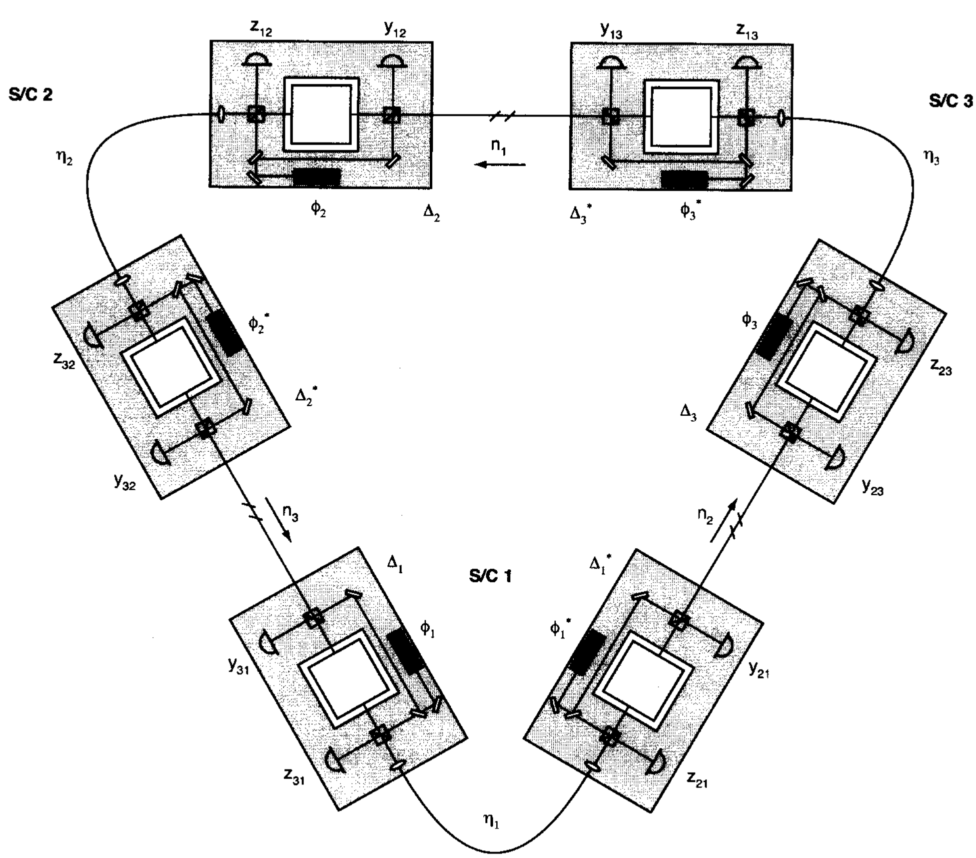

Figure 1: Optical layout of LISA interferometer showing six optical benches joined by three optical fibers and three free-space laser links.

This document contains a calculation for the LISA response when USO noise and doppler shifts are included. The bench noise and fiber noise is not included for clarity as it these noise sources

have been shown to cancel in a previous memo. The following assumptions are made:
1) Arm lengths are known exactly. 2) Laser frequencies, ν_i are slowly varying and the fluctuations are included as phase fluctuations.

$$\phi_i(t) = \nu_i t + p_i(t) \tag{1}$$

3) An ultra-stable oscillator (USO) is included on each space craft and is used as a master reference to measure the phase of all beatnotes. The USO phase is given by,

$$\phi_{uso(i)}(t) = f_i + q_i(t) \tag{2}$$

Each USO drives the clock input of a numerically controlled oscillator. The numerically controlled oscillator can give an output of $a_{ij}f_j + a_{ij}q_j(t)$. Notice that the f_j's are assumed to be slowly varying and all of the fluctuations are absorbed into the phases. From now on we will omit the time dependence of these variables and adopt the notation used in the Tinto, Armstrong, and Estabrook papers for representing delays.

The signals at phasemeter outputs associated with the 12 photodetectors are given by,

$$\begin{aligned}
y_{21} &= (\nu_1^* - \nu_3(1-\dot{L}_2) - a_{21}f_1)t + p_1^* - p_{3,2} - a_{21}q_1 + gw_{21} && (3)\\
z_{21} &= (\nu_1^* - \nu_1)t + p_1^* - p_1 - \eta_1 && (4)\\
y_{31} &= (\nu_1 - \nu_2^*(1-\dot{L}_3) - a_{31}f_1)t + p_1 - p_{2,3}^* - a_{31}q_1 + gw_{31} && (5)\\
z_{31} &= (\nu_1 - \nu_1^*)t + p_1 - p_1^* - \eta_1 && (6)
\end{aligned}$$

$$\begin{aligned}
y_{32} &= (\nu_2^* - \nu_1(1-\dot{L}_3) - a_{32}f_2)t + p_2^* - p_{1,3} - a_{32}q_2 + gw_{32} && (7)\\
z_{32} &= (\nu_2^* - \nu_2)t + p_2^* - p_2 - \eta_2 && (8)\\
y_{12} &= (\nu_2 - \nu_3^*(1-\dot{L}_1) - a_{12}f_2)t + p_2 - p_{3,1}^* - a_{12}q_2 + gw_{12} && (9)\\
z_{12} &= (\nu_2 - \nu_2^*)t + p_2 - p_2^* - \eta_2 && (10)
\end{aligned}$$

$$\begin{aligned}
y_{13} &= (\nu_3^* - \nu_2(1-\dot{L}_1) - a_{13}f_3)t + p_3^* - p_{2,1} - a_{13}q_3 + gw_{13} && (11)\\
z_{13} &= (\nu_3^* - \nu_3)t + p_3^* - p_3 - \eta_3 && (12)\\
y_{23} &= (\nu_3 - \nu_1^*(1-\dot{L}_2) - a_{23}f_3)t + p_3 - p_{1,2}^* - a_{23}q_3 + gw_{23} && (13)\\
z_{23} &= (\nu_3 - \nu_3^*)t + p_3 - p_3^* - \eta_3 && (14)
\end{aligned}$$

Note that we have include the effects of the doppler shift on the laser frequency, v_i (and ignored it's effects on the phase, q_i) of the beams from the distant spacecraft. Shot noise is currently not included although it should be transfered in the same manner as the GW signal. η_i is the fiber noise between benches on spacecraft i. In order to cancel this fiber noise it is necessary to only take combinations of $z_{xi} - z_{yi}$ at the same time. Thus we can combine the z_{ij}'s on the same spacecraft to form new variables $z_j = z_{xj} - z_{yj}$ and reduce the total number of data streams from twelve to nine.

$$\begin{aligned}
z_1 \equiv\ & z_{21} - z_{31} = && 2(\nu_1^* - \nu_1)t + p_1^* - p_1) && (15)\\
z_2 \equiv\ & z_{32} - z_{12} = && 2(\nu_2^* - \nu_2)t + p_2^* - p_2) && (16)\\
z_3 \equiv\ & z_{21} - z_{23} = && 2((\nu_3^* - \nu_3)t + p_3^* - p_3) && (17)
\end{aligned}$$

1 Phase locked transponders

If the lasers are phase locked in a coherent chain then the phase locking will drive various signals to zero. For the purposes of this discussion we define the phase locking hierarchy to be,

$$
\begin{aligned}
2 \rightarrow 2^* \rightarrow 1 \rightarrow 1^* && (18)\\
3^* \rightarrow 3 \rightarrow 1^* && (19)
\end{aligned}
$$

where the arrow points from slave to master laser. In this configuration laser 1^* is the ultimate master. These phase locking loops are enforced by feeding back to the appropriate laser phase (and thus frequency) to null the values of the following five observables.

$$
\begin{aligned}
z_1 &\rightarrow 0 \quad \text{via feedback to } \phi_1 && (20)\\
y_{32} &\rightarrow 0 \quad \text{via feedback to } \phi_2^* && (21)\\
z_2 &\rightarrow 0 \quad \text{via feedback to } \phi_2 && (22)\\
y_{23} &\rightarrow 0 \quad \text{via feedback to } \phi_3 && (23)\\
z_3 &\rightarrow 0 \quad \text{via feedback to } \phi_3^* && (24)\\
& && (25)
\end{aligned}
$$

Assuming there the lasers are phase locked with no frequency offset, ($a_{23} = a_{32} = 0$) the feedback imposes the following relationships between the phases of the slave lasers and the master laser, 1^*.

$$
\begin{aligned}
\nu_1 &= \nu_1^* && (26)\\
p_1 &= p_1^* && (27)\\
\nu_2 &= \nu_2^* = \nu_1^*(1 - \dot{L}_3) && (28)\\
p_2 &= p_2^* = p_{1,3}^* - gw_{32} && (29)\\
\nu_3 &= \nu_3^* = \nu_1^*(1 - \dot{L}_2) && (30)\\
p_3 &= p_3^* = p_{1,2}^* - gw_{23} && (31)
\end{aligned}
$$

In the absence of bench and fiber noise $\nu_i = \nu_i^*$ and $p_i = p_i^*$ and so for convenience we will use ν_i and p_i. The remaining four observables are recorded and processed to extract the gravitational wave signal.

$$
\begin{aligned}
y_{21} &= p_1 - p_{1,22} - a_{21}q_1 + gw_{21} + gw_{23,2} && (32)\\
y_{31} &= p_1 - p_{1,33} - a_{31}q_1 + gw_{31} + gw_{32,3} && (33)\\
y_{12} &= p_{1,3} - p_{1,21} - a_{12}q_2 + gw_{12} - gw_{32} + gw_{23,1} && (34)\\
y_{13} &= p_{1,2} - p_{1,31} - a_{13}q_3 + gw_{13} - gw_{23} + gw_{32,1} && (35)
\end{aligned}
$$

Notice that both laser and USO phase noise corrupt the phasemeter outputs. The coefficients of the USO noise terms are,

$$
\begin{aligned}
a_{21} &= \frac{2\nu_1 \dot{L}_2}{f_1} && (36)\\
a_{31} &= \frac{2\nu_1 \dot{L}_3}{f_1} && (37)\\
& && (38)
\end{aligned}
$$

2 Time Delay Interferometry

We can now take the usual time delay interferometry equations for the unequal-arm Michelson degrees of freedom X_1, X_2, and X_3 defined as,

$$
\begin{aligned}
X_1 &= y_{21} - y_{21,33} - y_{31} + y_{31,22} && (39)\\
X_2 &= y_{31,3} - y_{31,311} - y_{12} + y_{12,33} - y_{13,1} + y_{13,133} && (40)\\
X_3 &= y_{13} - y_{13,22} + y_{12,1} - y_{12,122} - y_{21,2} + y_{21,211} && (41)
\end{aligned}
$$

Considering X_1 only, and substituting from equations (34)-(35) we obtain,

$$
\begin{aligned}
X_1 &= p_1 - p_{1,22} - a_{21}q_1 + gw_{21} + gw_{23,2}\\
&\quad -p_{1,33} + p_{1,2233} + a_{21}q_{1,33} - gw_{21,33} - gw_{23,233}\\
&\quad -p_1 + p_{1,33} + a_{31}q_1 - gw_{31} - gw_{32,3}\\
&\quad +p_{1,22} - p_{1,3322} - a_{31}q_{1,22} + gw_{31,22} + gw_{32,322} && (42)\\
&= a_{31}(q_1 - q_{1,22}) - a_{21}(q_1 - q_{1,33})\\
&\quad +gw_{21} + gw_{23,2} - gw_{21,33} - gw_{23,233} - gw_{31} - gw_{32,3} + gw_{31,22} + gw_{32,322} && (43)
\end{aligned}
$$

Although the laser phase noise has exactly cancelled the clock noise remains in the final output.

3 Auxiliary laser beams

It has been suggested elsewhere that the clock noise can be calibrated if a second set of laser links is used. These laser links are envisaged to be phase locked to the main laser with a frequency offset by the local USO. The interference between the local auxiliary laser and the distant auxiliary laser is measured and recorded. The phase meter outputs for the measurement of these secondary lasers are given below.

$$
\begin{aligned}
y'_{21} &= (\nu_1 + f_1 - (\nu_3 + f_3)(1 - \dot{L}_2) - b_{21}f_1)t + p_1 - p_{3,2} + (1 - b_{21})q_1 - q_{3,2} + gw_{21} && (44)\\
y'_{31} &= (\nu_1 + f_1 - (\nu_2 + f_2)(1 - \dot{L}_3) - b_{31}f_1)t + p_1 - p_{2,3} + (1 - b_{31})q_1 - q_{2,3} + gw_{31} && (45)
\end{aligned}
$$

$$
\begin{aligned}
y'_{32} &= (\nu_2 + f_2 - (\nu_1 + f_1)(1 - \dot{L}_3) - b_{32}f_2)t + p_2 - p_{1,3} + (1 - b_{32})q_2 - q_{1,3} + gw_{32} && (46)\\
y'_{12} &= (\nu_2 + f_2 - (\nu_3 + f_3)(1 - \dot{L}_1) - b_{12}f_2)t + p_2 - p_{3,1} + (1 - b_{12})q_2 - q_{3,1} + gw_{12} && (47)
\end{aligned}
$$

$$
\begin{aligned}
y'_{13} &= (\nu_3 + f_3 - (\nu_2 + f_2)(1 - \dot{L}_1) - b_{13}f_3)t + p_3 - p_{2,1} + (1 - b_{13})q_3 - q_{2,1} + gw_{13} && (48)\\
y'_{23} &= (\nu_3 + f_3 - (\nu_1 + f_1)(1 - \dot{L}_2) - b_{23}f_3)t + p_3 - p_{1,2} + (1 - b_{23})q_3 - q_{1,2} + gw_{23} && (49)
\end{aligned}
$$

Assuming the carrier lasers are phase locked as described earlier the sideband-sideband beat at space-crafts 3 and 2 become,

$$
\begin{aligned}
y'_{23} &= (f_3 - f_1(1 - \dot{L}_2) - b_{23}f_3)t + (1 - b_{23})q_3 - q_{1,2} && (50)\\
y'_{32} &= (f_2 - f_1(1 - \dot{L}_3) - b_{32}f_2)t + (1 - b_{32})q_2 - q_{1,3} && (51)
\end{aligned}
$$

4 Phaselocking USOs

Just as we obtained a significant improvement (in terms of the number of data streams required) when we perform two-way measurements rather than one-way measurements for the carriers, we can also implement two-way measurements on the USO noise calibration. We can drive these sideband-sideband beatnotes' phases to zero by feeding back to f_3, f_2, p_3, and p_2 effectively phase locking USOs 3 and 2 to USO 1. Assuming no offset in the phase locking (ie $b_{23} = b_{32} = 0$). This enforces the following conditions,

$$\begin{aligned} f_3 &= f_1(1 - \dot{L}_2) && (52) \\ f_2 &= f_1(1 - \dot{L}_3) && (53) \\ q_3 &= q_{1,2} && (54) \\ q_2 &= q_{1,3} && (55) \end{aligned}$$

With this phase locking (for both the carrier and the sideband) in operation the phase of the interference back at space-craft 1 becomes,

$$\begin{aligned} y'_{21} &= y_{21} + (q_1 - q_{1,22}) + 2\dot{L}_2 q_1 && (56) \\ y'_{31} &= y_{31} + (q_1 - q_{1,33}) + 2\dot{L}_3 q_1 && (57) \end{aligned}$$

The term $2\dot{L}_2 q_1$ can be ignored for state of the art USOs and doppler shifts of 10 MHz or less. We can now form a combination that is free from USO noise and contains the same gravitational wave signal by taking,

$$\begin{aligned} X'_1 &= X_1 - a_{31}(y'_{21} - y_{21}) + a_{21}(y'31 - y_{31}) && (58) \\ &= gw_{21} + gw_{23,2} - gw_{21,33} - gw_{23,233} - gw_{31} - gw_{32,3} + gw_{31,22} + gw_{32,322} && (59) \end{aligned}$$

Note that if we make a_{31} and a_{21} equal unity and choose the correct sign then it may be possible to cancel the terms y_{21} and the final expression for X'_1 that is free from laser noise, bench noise and USO noise is simply,

$$X'_1 = y'_{21} - y_{21,33} - y'_{31} + y_{31,22} \qquad (60)$$

To implement this idea it would require the following steps.

1) Turn on laser 1.
2) Phase lock laser 2 (and 3) to laser 1 at far s/c.
3) Detect the interference beatnote at 2 times the doppler frequency (y_{21} and y_{31}).
4) Set (or lock weakly with a feedback time constant of several hours) each of the numerically controlled oscillators (NCOs) to those doppler frequencies.
5) Phase lock the auxiliary lasers to the carriers on s/c 1 with a frequency offset equal to the NCO frequencies.
6) Phase lock spacecraft 2 (and 3) auxiliary lasers to the auxiliary lasers from s/c 1 received at s/c 2 (and s/c 3).
7) Measure the interference between the 2 lasers back at s/c 1 (y'_{21} and y'_{31}).